```
<< C:\drorbn\AcademicPensieve\Projects\Mathematica\LinAlg.m

Rerouting[n_] := Module[
   {P1, P2, rrt},
   rrt = Total[(Signature[#] * (P1@#)) & /@ Permutations[Range[n]]];
   Expand[rrt * (rrt /. P1 → P2)] /. P1[p1_] P2[p2_] :→ (
      P @@ (p1 ~ Join ~ (p2 + n)) - P @@ ((p1 + n) ~ Join ~ p2)
     )
  ];
Lambda[m_, n_, auxins_List, auxouts_List] := Module[
   {basic, ins, outs},
   ins = Complement[Range[m], auxins];
   outs = Complement[Range[m], auxouts];
   basic = P @@ Range[m];
   basic[[auxins]] = auxouts;
   basic[[ins]] = outs;
   Total[
     (basic /. Thread[outs → #] & /@ Permutations[outs]) /.
      p_P :→ Signature[p] * p
   ]
  ];
AllLambdas[m_, n_] := Flatten[Outer[
   Lambda[m, n, #1, #2] &,
   Subsets[Range[m], {m - n}],
   Permutations[Range[m], {m - n}],
   1
  ]]

Rerouting[2]

P[1, 2, 3, 4] - P[1, 2, 4, 3] - P[2, 1, 3, 4] + P[2, 1, 4, 3] -
 P[3, 4, 1, 2] + P[3, 4, 2, 1] + P[4, 3, 1, 2] - P[4, 3, 2, 1]

Lambda[6, 4, {3, 5}, {5, 2}]

P[1, 3, 5, 4, 2, 6] - P[1, 3, 5, 6, 2, 4] - P[1, 4, 5, 3, 2, 6] + P[1, 4, 5, 6, 2, 3] +
 P[1, 6, 5, 3, 2, 4] - P[1, 6, 5, 4, 2, 3] - P[3, 1, 5, 4, 2, 6] + P[3, 1, 5, 6, 2, 4] +
 P[3, 4, 5, 1, 2, 6] - P[3, 4, 5, 6, 2, 1] - P[3, 6, 5, 1, 2, 4] + P[3, 6, 5, 4, 2, 1] +
 P[4, 1, 5, 3, 2, 6] - P[4, 1, 5, 6, 2, 3] - P[4, 3, 5, 1, 2, 6] + P[4, 3, 5, 6, 2, 1] +
 P[4, 6, 5, 1, 2, 3] - P[4, 6, 5, 3, 2, 1] - P[6, 1, 5, 3, 2, 4] + P[6, 1, 5, 4, 2, 3] +
 P[6, 3, 5, 1, 2, 4] - P[6, 3, 5, 4, 2, 1] - P[6, 4, 5, 1, 2, 3] + P[6, 4, 5, 3, 2, 1]

AllPs[m_] := (P @@ #) & /@ Permutations[Range[m]];

LC2SA[AllPs[6], Lambda[6, 4, {3, 5}, {5, 2}]]

SparseArray[<24>, {720}]

Subsets[Range[4], {3}]

{{1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}}
```

```
AllLambdas[4, 3]
```

```
{P[1, 2, 3, 4] - P[1, 2, 4, 3] - P[1, 3, 2, 4] + P[1, 3, 4, 2] + P[1, 4, 2, 3] - P[1, 4, 3, 2],
 -P[2, 1, 3, 4] + P[2, 1, 4, 3] + P[2, 3, 1, 4] - P[2, 3, 4, 1] - P[2, 4, 1, 3] + P[2, 4, 3, 1],
 P[3, 1, 2, 4] - P[3, 1, 4, 2] - P[3, 2, 1, 4] + P[3, 2, 4, 1] + P[3, 4, 1, 2] - P[3, 4, 2, 1],
 -P[4, 1, 2, 3] + P[4, 1, 3, 2] + P[4, 2, 1, 3] - P[4, 2, 3, 1] - P[4, 3, 1, 2] + P[4, 3, 2, 1],
 -P[2, 1, 3, 4] + P[2, 1, 4, 3] + P[3, 1, 2, 4] - P[3, 1, 4, 2] - P[4, 1, 2, 3] + P[4, 1, 3, 2],
 P[1, 2, 3, 4] - P[1, 2, 4, 3] - P[3, 2, 1, 4] + P[3, 2, 4, 1] + P[4, 2, 1, 3] - P[4, 2, 3, 1],
 -P[1, 3, 2, 4] + P[1, 3, 4, 2] + P[2, 3, 1, 4] - P[2, 3, 4, 1] - P[4, 3, 1, 2] + P[4, 3, 2, 1],
 P[1, 4, 2, 3] - P[1, 4, 3, 2] - P[2, 4, 1, 3] + P[2, 4, 3, 1] + P[3, 4, 1, 2] - P[3, 4, 2, 1],
 P[2, 3, 1, 4] - P[2, 4, 1, 3] - P[3, 2, 1, 4] + P[3, 4, 1, 2] + P[4, 2, 1, 3] - P[4, 3, 1, 2],
 -P[1, 3, 2, 4] + P[1, 4, 2, 3] + P[3, 1, 2, 4] - P[3, 4, 2, 1] - P[4, 1, 2, 3] + P[4, 3, 2, 1],
 P[1, 2, 3, 4] - P[1, 4, 3, 2] - P[2, 1, 3, 4] + P[2, 4, 3, 1] + P[4, 1, 3, 2] - P[4, 2, 3, 1],
 -P[1, 2, 4, 3] + P[1, 3, 4, 2] + P[2, 1, 4, 3] - P[2, 3, 4, 1] - P[3, 1, 4, 2] + P[3, 2, 4, 1],
 -P[2, 3, 4, 1] + P[2, 4, 3, 1] + P[3, 2, 4, 1] - P[3, 4, 2, 1] - P[4, 2, 3, 1] + P[4, 3, 2, 1],
 P[1, 3, 4, 2] - P[1, 4, 3, 2] - P[3, 1, 4, 2] + P[3, 4, 1, 2] + P[4, 1, 3, 2] - P[4, 3, 1, 2],
 -P[1, 2, 4, 3] + P[1, 4, 2, 3] + P[2, 1, 4, 3] - P[2, 4, 1, 3] - P[4, 1, 2, 3] + P[4, 2, 1, 3],
 P[1, 2, 3, 4] - P[1, 3, 2, 4] - P[2, 1, 3, 4] + P[2, 3, 1, 4] + P[3, 1, 2, 4] - P[3, 2, 1, 4]}
```

```
AllLambdas2 = Flatten[Table[
   Lambda[4, 3, {i}, {j}],
   {i, 4}, {j, 4}
  ]]
```

```
{P[1, 2, 3, 4] - P[1, 2, 4, 3] - P[1, 3, 2, 4] + P[1, 3, 4, 2] + P[1, 4, 2, 3] - P[1, 4, 3, 2],
 -P[2, 1, 3, 4] + P[2, 1, 4, 3] + P[2, 3, 1, 4] - P[2, 3, 4, 1] - P[2, 4, 1, 3] + P[2, 4, 3, 1],
 P[3, 1, 2, 4] - P[3, 1, 4, 2] - P[3, 2, 1, 4] + P[3, 2, 4, 1] + P[3, 4, 1, 2] - P[3, 4, 2, 1],
 -P[4, 1, 2, 3] + P[4, 1, 3, 2] + P[4, 2, 1, 3] - P[4, 2, 3, 1] - P[4, 3, 1, 2] + P[4, 3, 2, 1],
 -P[2, 1, 3, 4] + P[2, 1, 4, 3] + P[3, 1, 2, 4] - P[3, 1, 4, 2] - P[4, 1, 2, 3] + P[4, 1, 3, 2],
 P[1, 2, 3, 4] - P[1, 2, 4, 3] - P[3, 2, 1, 4] + P[3, 2, 4, 1] + P[4, 2, 1, 3] - P[4, 2, 3, 1],
 -P[1, 3, 2, 4] + P[1, 3, 4, 2] + P[2, 3, 1, 4] - P[2, 3, 4, 1] - P[4, 3, 1, 2] + P[4, 3, 2, 1],
 P[1, 4, 2, 3] - P[1, 4, 3, 2] - P[2, 4, 1, 3] + P[2, 4, 3, 1] + P[3, 4, 1, 2] - P[3, 4, 2, 1],
 P[2, 3, 1, 4] - P[2, 4, 1, 3] - P[3, 2, 1, 4] + P[3, 4, 1, 2] + P[4, 2, 1, 3] - P[4, 3, 1, 2],
 -P[1, 3, 2, 4] + P[1, 4, 2, 3] + P[3, 1, 2, 4] - P[3, 4, 2, 1] - P[4, 1, 2, 3] + P[4, 3, 2, 1],
 P[1, 2, 3, 4] - P[1, 4, 3, 2] - P[2, 1, 3, 4] + P[2, 4, 3, 1] + P[4, 1, 3, 2] - P[4, 2, 3, 1],
 -P[1, 2, 4, 3] + P[1, 3, 4, 2] + P[2, 1, 4, 3] - P[2, 3, 4, 1] - P[3, 1, 4, 2] + P[3, 2, 4, 1],
 -P[2, 3, 4, 1] + P[2, 4, 3, 1] + P[3, 2, 4, 1] - P[3, 4, 2, 1] - P[4, 2, 3, 1] + P[4, 3, 2, 1],
 P[1, 3, 4, 2] - P[1, 4, 3, 2] - P[3, 1, 4, 2] + P[3, 4, 1, 2] + P[4, 1, 3, 2] - P[4, 3, 1, 2],
 -P[1, 2, 4, 3] + P[1, 4, 2, 3] + P[2, 1, 4, 3] - P[2, 4, 1, 3] - P[4, 1, 2, 3] + P[4, 2, 1, 3],
 P[1, 2, 3, 4] - P[1, 3, 2, 4] - P[2, 1, 3, 4] + P[2, 3, 1, 4] + P[3, 1, 2, 4] - P[3, 2, 1, 4]}
```

```
List[
 LC2SA[AllPs[4], AllLambdas2],
 LC2SA[AllPs[4], Rerouting[2]]
]
```

```
{SparseArray[<96>, {16, 24}], SparseArray[<8>, {24}]}
```

```
MatrixRank[LC2SA[AllPs[4], AllLambdas2]]
```

```
10
```

```
LinearSolve[
 Transpose[LC2SA[AllPs[4], AllLambdas2]],
 LC2SA[AllPs[4], Rerouting[2]]
]
```

{0, 0, -1, -1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
NullSpace[LC2SA[AllPs[4], AllLambdas2] // Transpose]
```

{{1, 1, 1, 2, -1, -1, -1, 0, -1, -1, -1, 0, 0, 0, 0, 1},
 {-1, -1, 0, -1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0},
 {-1, 0, -1, -1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0},
 {0, -1, -1, -1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0},
 {-1, -1, -1, -1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0},
 {-1, -1, -1, -1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0}}

```
LinearSolve[
 Transpose[LC2SA[AllPs[6], AllLambdas[6, 4]]],
 LC2SA[AllPs[6], Rerouting[3]]
]
```

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1,
 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, -1, -1, 0, 0, -1, -1, -1, 0, 0,
 -1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```
LinearSolve[
 Transpose[LC2SA[AllPs[8], AllLambdas[8, 5]]],
 LC2SA[AllPs[8], Rerouting[4]]
]
```